§6-6 Optimization Problems III: Linear Programming

Linear Programming - A mathematical technique used to determine which solutions in the feasible region result in the optimal solutions of the objective function.

Example 2 (p338)

Let x be the number of narrow boards.
y be the number of wide boards.
C be the cost of the lumber

Restrictions

$x \in W$ and $y \in W$

Constraints

$x \geq 0$
$y \geq 0$] * not necessary since whole numbers are ≥ 0

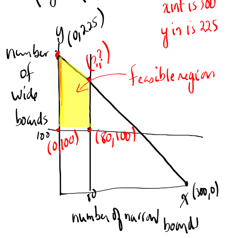
$x \leq 80$

$y \geq 100$

$6x + 8y \leq 1800$

$50\,yd\left(\frac{3\,ft}{yd}\right)\left(\frac{12\,in}{1\,ft}\right) = 1800$ inches

x int is 300
y int is 225



Objective Function

$C = 3.56x + 4.36y$

Find the intersection of $x = 80$ and $6x + 8y = 1800$

$6(80) + 8y = 1800$
$480 + 8y = 1800$
$8y = 1320$
$y = 165$

The intersection pt or vertex will be (80, 165)

Vertex	$C = 3.56x + 4.36y$	Cost
* (0, 100)	$3.56(0) + 4.36(100)$	\$436 minimum
(0, 225)	$3.56(0) + 4.36(225)$	\$981
(80, 100)	$3.56(80) + 4.36(100)$	\$720.80
* (80, 165)	$3.56(80) + 4.36(165)$	\$1004.20 maximum

Verify the optimal points (solutions) and check that they satisfy the constraints.

(0, 100) ✓ Check both solutions.

(80, 165) ✓

Final Answer:

A combination of 0 narrow boards and 100 wide boards would cost the least (\$436)

A combination of 80 narrow boards and 165 wide boards would cost the most (\$1004.20)

TO DO

1. CYU (p341-342)
2. Practise (p343/5-15)

Need to Know

- The solution to an optimization problem is usually found at one of the vertices of the feasible region.
- To determine the optimal solution to an optimization problem using linear programming, follow these steps:

  Step 1. Create an algebraic model that includes:
  - a defining statement of the variables used in your model
  - the restrictions on the variables
  - a system of linear inequalities that describes the constraints
  - an objective function that shows how the variables are related to the quantity to be optimized

  Step 2. Graph the system of inequalities to determine the coordinates of the vertices of its feasible region.

  Step 3. Evaluate the objective function by substituting the values of the coordinates of each vertex.

  Step 4. Compare the results and choose the desired solution.

  Step 5. Verify that the solution(s) satisfies the constraints of the problem situation.